Discrete Mathematics I

B. Math. II

Semestral Examination

Instructions: All questions carry ten marks. All graphs are assumed to be simple.

1. For a graph G, a matching with maximum number of edges is called a *maximum* matching. Give an example of a maximal matching of a graph G that is not a maximum matching. However, prove that if W is the set of vertices of a matching, then there exists a maximum matching whose vertices contain W.

2. For a natural number m, determine the number of vertices of a tree which has exactly one vertex of degree i for every $2 \leq i \leq m$ and all other vertices having degree 1.

3. Let O be a subset of a projective plane of order n such that no three points of O are collinear. Prove that $|O| \leq n+2$ and equality holds only if n is even.

4. Define Steiner triple systems. Prove that if it exists on v points then $v-1$ or $v-3$ must be divisible by 6.

5. Prove that every t-design is also an i-design for every $0 \leq i \leq t$.

6. Let A be a partial Latin square of order n in which (i,j) th entry is filled if and only if $i \leq r$ and $j \leq s$. Give a necessary and sufficient condition for A to be completed to a Latin square of order n.